Interpreting Distance Graphs

Updated: 05/15/10

Objective:
Students will recognize the characteristics of a distance graph.

Connections to Previous Learning:
Students should understand the relationship between distance and time.

Connections to AP*:
AP Calculus Topics: Analysis of Functions; Rate of Change

Materials:
Student activity pages

Teacher Notes:
To introduce the idea that the distance graph is not the path being walked, use the following scenarios before opening the lesson pages.

Scenario 1: Ask a student to walk 8 feet across the front of the room moving along a horizontal line and then to return to the starting position. Ask the class to draw a picture of the student’s path. It should be a horizontal line segment, 8 units long.

Scenario 2: Ask a student to walk across the room as you read the following set of instructions. These instructions model the walk described in the lesson.

1. From your starting point, walk 3 feet along a horizontal line, moving at a constant rate.
2. Walk 1 foot more at a constant rate that is slower than your original rate.
3. Walking at a rate that is faster than your original rate, go forward 4 feet.
4. Turn around and walk 3 feet back toward your starting point at the original constant rate.
5. Stop to rest.
6. Walk 3 feet toward the starting point at the original constant rate.
7. Walk 2 feet to return to the original position at a slower constant rate.

Again, ask the class to draw a picture of the student’s path. The picture will be exactly the same as the one drawn in scenario 1.

The distance-time graph for Ann’s walk shown in the lesson indicates how far Ann is from home at a given time. The independent variable for the graph is time and not distance. Her distance is from home measured along the vertical axis.

At the middle grades level when a function is given on a grid, points at the intersection of two grid lines can be read from the graph. If the grid is not provided, only labeled points should be read from the graph. All other points should be determined by analytical methods.
Interpreting Distance Graphs

Ann went for a walk on Saturday. When the timing started, she was already traveling at the given rate. She walked in a straight line away from home, then she returned back along the same path. On her way home, she stopped for lunch. The graph shows her distance from home at any given time during the walk. Complete the table, and then use the graph and table to answer the following questions.

1. At what time is Ann’s distance from home 0 miles? Give a reason that supports your answer.

2. What is Ann’s maximum distance from home? At what time did she turn around to return home? Give reasons that support your answers.

3. When did Ann stop for lunch? How far from home was she when she stopped for lunch? How long did she stop for lunch? Give reasons that support your answers.

4. How many hours was Ann’s trip? Give a reason that supports your answer.

5. When did Ann walk the fastest? What was her speed in miles per hour? Give reasons that support your answers.
6. When was Ann walking the slowest? What was her speed in miles per hour? Give reasons that support your answers.

7. What was Ann’s distance from home at 6 hours? Give a reason that supports your answer.

8. At what times was Ann’s distance from home 4 miles? (Hint: use similar triangles to find the second time.)

9. During what times was Ann’s distance from home increasing? Give a reason that supports your answer.

10. During what times was Ann’s distance from home decreasing? Give a reason that supports your answer.

11. When was Ann walking the same rate away from home and toward home? Give a reason that supports your answer.
Interpreting Distance Graphs

**Answers:**
*After each answer the question is rewritten for a function f(x).*

<table>
<thead>
<tr>
<th>Time in hours</th>
<th>Distance from home in miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

1. Ann’s distance from home was 0 miles at 0 and 7 hours because these are the values of \( t \) where the distance is 0.
   *What are the \( t \)-intercepts of the function?*

2. Ann’s maximum distance from home was 8 miles because this is the highest \( y \)-value on the graph. She has reached the maximum distance at 3 hours because this is the \( x \)-value that corresponds to the highest \( y \)-value.
   *What is the maximum value of the function?*

3. After 4 hours, when she was 5 miles from home, Ann stopped for one hour for lunch. There is a horizontal line on the graph which indicates the distance is not changing as the hour passes.
   *When was the rate of change 0?*

4. Ann’s trip was 7 hours. At \( t = 7 \), the distance is 0 miles from home.
   *What is the domain of the function?*

5. Between 2 and 3 hours Ann walked 4 miles per hour. In this interval, the line is the steepest which means the slope of the line is the greatest of all those on the graph. The slope of the line represents change in distance divided by change in time.
   *What is the maximum rate of change?*

6. Between 1 and 2 hours Ann walked 1 mile per hour. In this interval, the line is the least steep which means the slope of the line is the smallest of all those on the graph with the exception of the horizontal line between 4 and 5 hours when she stopped for lunch. The slope of the line represents change in distance divided by change in time.
   *What is the minimum rate of change?*

7. After 6 hours Ann was 2 miles from home. The point (6, 2) is a point on the graph.
   *What is the value of \( f(6) \)?*
8. Ann was 4 miles from home at 2 hours and at $5\frac{1}{3}$ hours. These are values for $t$ where the graph of $y = 4$ would intersect the distance graph.

To find the value $5\frac{1}{3}$, use $\triangle ADE \sim \triangle ABC$, so $\frac{CB}{1} = \frac{1}{3}$.

$5 + CB = 5\frac{1}{3}$ so point B is located at $\left(5\frac{1}{3}, 4\right)$.

What is the value of $t$ when $f(t) = 4$?

9. Ann’s distance from home was increasing between 0 and 3 hours because the graph is increasing (as the $t$ values increase, the corresponding $y$-values increase).

What is the interval where $f(t)$ is increasing?

10. Ann’s distance from home was decreasing between 3 and 4 hours and between 5 and 7 hours because the graph is decreasing (as the $t$ values increase, the corresponding $y$-values decrease).

What are the intervals where $f(t)$ is decreasing?

11. Ann was walking away from home between 0 and 1 hour at the same speed as when she was walking towards home between 3 and 4 hours and 5 and 6 hours because the slopes have the same absolute value.

When are the absolute values of the rates of change equal?